D5.1 Report with review of balancing routines

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Content

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1. Introduction

The rapid increase in population and economic activity are signs that we challenging the supply of resources in the near future. Over the last decades, especially Asia-pacific experienced an exceptional economic growth, which makes this region one of the major drivers towards overshooting global resource use limits (Schandl and West (2010)). While these are the regions where most production takes place, also more developed regions take their share in responsibility of resource scarcity due to increased welfare standards. The climate discussion is high on the agenda of many countries worldwide. The goal is to protect our planet and limit temperature increase, but we should not forget that it also puts pressure on the use of resources.

The European Commission adopted the Raw Material Initiative, which should tackle the issue of access to raw materials in the EU (European Commission, 2019). Policies that support these efforts in resource efficiency should be supported by detailed data and research on this topic. Surprisingly, there is not yet a database that maps the flows and stocks of materials. The goal of the PANORAMA project is to create such a database for EIT KIC Raw Materials. While other work packages gather data (WP3), process the data and fill in the blanks (WP4), WP5 is responsible for balancing of the database.

There is a large stream of literature that deals with the issue of balancing a matrix. Each of these methods have their own advantages and disadvantages. Most balancing procedures are developed for square (monetary) databases. However, the database developed in Panorama is not two-dimensional but three-dimensional. Either a new method should be developed for balancing this database or one of the existing methods requires an update. In order to choose an appropriate balancing procedure, we first make a thorough review of various methods (Appendix 1).

This report provides a literature review of the existing balancing procedures and indicates which valuable properties hold for each approach. Section 2.1. gives a description of the desired balancing properties and Section 2.2 briefly introduces the different balancing procedures mentioned in the literature review and Section 3 concludes.

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2. Summary of literature review

This section presents an overview of the most prominent balancing procedures used for Input-output tables, which serve as an inspiration for the methodology to be applied in for the PANORAMA database. Previous literature highlights the importance and contribution of specific balancing procedures. We compare all these balancing procedures based on a set of criteria in order to assess their strenghts and weaknessess in terms of the PANORAMA database requirements.

2.1. Balancing properties

For balancing a square matrix, row sums should be equal to the column sums. The literature review in Appendix A describes eleven balancing procedures which all have their own advantages and disadvantages. In order to categorize the advantages of these models, six properties have been defined.

The balancing proporties against which the balancing procedures are held for comparison are:

- Incorporate constraints on arbitrarily sized and shaped subsets of matrix elements, instead of only fixing row and column sums;
- allow considering the reliability of the initial estimate;
- be able to handle negative values and to preserve the sign of matrix elements if required;
- be able to handle conflicting external data;
- uses limited computational effort;
- is able to balance a three-dimensiontal table

Lenzen et al. (2009) already distinguished five balancing properties. The first four balancing properties presented above are taken from Lenzen et al. (2009). The latter two, limited computational effort and the three-dimensional element are added by us. Since the Panorama database will be a very large database, with at least 200 products, 163 industries and 49 regions the efficiency of balancing becomes important. Also, as mentioned earlier, the balancing procedures are defined for two-dimensional tables. Panorama needs to balance a three-dimensional table.

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2.2. Balancing procedures

Eleven prominent balancing procedures are chosen for the literature review of balancing procedures, where we make the distinction between iterative and optimization methods.

Iterative approaches

RAS is probably the best known iterative balancing procedure. Besides RAS, five modified versions of the iterative RAS approach are included in the literature review: MRAS, ERAS, TRAS, GRAS, KRAS. These modifications are given in order of development in time. In general, each later modification is an improvement of the last modified approach.

In summary, MRAS stands for modified RAS. Contrary to RAS, it takes the reliability of initial estimates into account by assuming that some elements of the matrix are certain and cannot be changed. The RAS procedures is performed on the remaining elements of the matrix. Also ERAS allows to fix interior values of the forecasted matrix to certain values. TRAS stands for three-staged RAS approach. It includes an extra step in the iterative procedure where the balancing procedure should also satisfy constraints on aggregated subsets of the matrix. GRAS, a generalized RAS approach deals with negative values in a square matrix. Rather then assuming that negative values are fixed and taking out of the balancing procedure, GRAS proposes to use absolute values for negative elements in the objective function. Lastly, KRAS stands for Konfliktfreies RAS approach. It extends GRAS by giving a solution for conflicing external informatino and inconsistent constraints.

For each of these RAS based approach, the step-wise iterative procedure has been presented in boxes in the text.

Constrained optimization approaches

Besides iterative methods, also constrained optimization approaches aim at solving balancing problems. An objective function is minimized under a set of constraints. Most optimization approach own their name to the type of objective function.

Five contrained optimization approaches have been defined in this document: Maximum entropy (ME), cross-entropy (CE), general cross-entropy (GCE), (generalized) least-square (LS) and the linear method (LM). The formulations of objective functions of the respective approaches can be found in Appendix A. This section only highlights the key qualities of each approach.

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The main difference between the ME and the CE approach is that CE takes prior information into account via the initial estimates. This prior information could be taken fom a database of a previous year. ME assumes that each element of a m by m matrix is initially equal to $1/m$. Both approaches can be extened by including more information in the constraints. That is, including information on economic aggregates or uncertainty in aggregates, or keeping zeros in the initial matrix fixed to zero.

GCE is a variation of the CE approach. CE treats elements of a matrix as probabilities, and the total matrix is considered the probability distribution. GCE on the other hand, treats each individual element of a matrix as a random variable that can take a range of values, i.e. the support vector. This support vector is connected to a symmetric probability vector. In the balancing procedure, the support vector is assumed given and the corresponding vector of probabilities is optimized. The expected value that results from optimized probabilities gives the best estimate of the matrix element.

The LS method assumes to minimize the square distance between the best estimate and the initial estimate. Also LS can be extended by including more constraints. The extention given in the literature review additionally makes the distinction between hard constraints and soft constraints. It thereby takes into account the reliability of the different contraints. The linear method is a deviation of the LS method. That is, it makes use of the analytical solution of the LS method. This allows the linear method to iteratively update the matrix, which benefits the computational effort for balancing a matrix.

3. Conclusion

Eleven balancing procedures are discussed in the literature review. These balancing procedures are compared using a selection of desirable properties. This comparison yielded three methods that satisfy most of these properties: KRAS, generalized Least Squares approach, and the linear approach.

For the PANORAMA database we chose to use the Linear approach for 3 reasons: (1) it belongs to one fo the procedures that has most desirable properties, (2) contrary to GLS, the linear approach is solved in an iterative manner which requires less computational power, (3) the founder of this approach is a member of the PANORAMA team, facilitating any required adjustments for a 4-dimensional matrix.

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4. References

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5. Appendix A

This Appendix presenting the full literature review is given below.

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Appendix A: Literature review on balancing methods for the PANORAMA project

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1 Introduction

The transitions toward a low-carbon, circular economy requires a global understanding of commodity production and consumption, including the emissions and waste throughout its supply chain, and its fate after its lifetime. All this information needs to be integrated into a physical stock-flow system that can ascertain future economy demands together with the pressure on raw material extraction. This information can be represented in (environmentally-extended) supply-use and input-output tables (SUT and IOT, respectively), which can give insight into the connection between economic development, human development and changes in environment (Pauliuk et al., 2015). Much work has been done regarding collection of physical databases (e.g. MICA, PROSUM, Minerals4EU, or DESIRE). However, a comprehensive, consistent and balanced supply chain and stock-flow materials database for many materials is still missing. The PANORAMA project aims to fills this gap by creating such a database.

Similar systems have been developed for monetary flows. For instance, EXIOBASE is a monetary multi-regional input-output database that has been developed and updated in three H2020 programs: EXIOPOL, CREEA and DESIRE (see Tukker et al. (2009), Wood et al. (2015) and Merciai and Schmidt (2016)). These databases use national (e.g. supply-use tables) and international statistics (trade, global production) to represent the global economy. Typically, various nations report sectors, products, and imports/exports in different units, systems, and catergories. Collecting data from different sources thus results initially in an incomplete, inconsistent and unbalanced database. While an initial level of harmonization is required bin order to standardize the various data sources, the database will likely remain undetermined. For this reason, imputation and balancing procedures are necessary to come to a coherent database, and a comprehensive set used in supply-use and input-output systems is presented in this review.

This document is constructed as follows: Section 2 introduces some concepts that should be clear before starting to describe different balancing procedures. That is, the structure of the to be balanced database, the importance of imputation before starting the balancing procedure and last, the most important properties that a balancing procedure should fulfil. The literature review is given in Section 3. Given the description and properties of the existing methods, in Section 3.3 an overview of all methods is given, based on the properties in Section 2.3. The literature review aims to accommodate the decision for a balancing procedure in the panorama project.

2 Clarification of basic concepts

Economic balancing routines are developed for monetary databases. In general, the 'monetary database' refers to an input-output table or a social accounting matrix (SAM). Section 2.1 describes the structure of a SAM and the economic meaning of the underlying equations when row sums should

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be equal to column sums. Section 2.2 highlights the importance of imputation. The imputation problem should be dealt with before starting the balancing procedure. Finally, Section 2.3 explains the desirable properties for balancing procedures.

2.1 Structure of a Social Accounting Matrix

Most balancing routines are developed for monetary databases, while PANORAMA involves a physical table. In the literature review we refer to a Social Accounting Matrix (SAM), a monetary database. However, a SAM can also be created for physical supply and use tables, and thus the approach is similar. This section explains the structure of a monetary SAM.

The structure of a SAM is given in Table 2.1, after Miller and Blair (2009). The definition of the matrices that are part of the SAM is given in Table 2.1. A SAM is a square matrix that is balanced when the row rums equal the column sums. This can be given for each row/column element in Table 2.1.

Consumption account: $Q + M + T_I = U + I + X + G + F$.
Total consumed equals total produced. That is, total value consumed by industries (U), Capital accumulation (I), Rest of the World regions (E), Government (G) and Households (F), should be produced somewhere in the economy. This is either produced by industries in regions that are included in the supply table (Q), by industries in regions not included in supply table (M), or otherwise, it goes into import taxes (T_I).

Production account: $U + T_B + V = Q + D + H$.
Spendings of industries equal earnings of industries. That is, industries spend value on purchasing intermediate goods (U), paying taxes (T_B) and other capital and labor expenses in the value added category (V). Industries earn money by selling products and services (Q), the value of capital goods owned by industries (D), and by incomes that are generated in RoW regions (H).

Capital accumulation account: $I + D + L + B = S_F + S_G + S$.
Total savings equal total investments. Savings are coming from households (H), government (G) and from regions that are not part of supply and use tables (F). Investments are made on value of owned commodities (I), reduced by depreciation of capital (D), investments can be lending to RoW regions (L) or to domestic governments (B).

Balance of payments account: $X + H + S_F = L + M + O$.
Total earned from foreign regions equals total spend on foreign regions. Earned value comes from export of products to foreign regions (X), Income generated in foreign regions (H), and savings of foreign regions (S).

Goverment account: $G + S_G + P = T_I + T_B + B + T$.
Total earned by governments equals total spend by government. Earnings are coming from tax incomes. Governments spend money on commodity goods (G), savings (S_G) and social welfare services (P).

Household accounts: $F + S + O + T = P + W$.
Total earned by households equals total spend by households. Earnings are coming from wages (W) and social welfare services of the government (P). Household dedicate this income on purchase of commodities and services (F), savings (S), they spend it in foreign regions (O), or on (income) taxes (T).

Value added accounts: $V = W$.
Total use of value added by industries equals total income generated by households.

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Table 1: Structure of Social Accounting matrix (Miller and Blair (2009))

	Com.	Ind.	Cap.	ROW	Govt.	HH	VA
Commodities		U	I	X	G	F	
Industries	Q		D	H			
Capital accumulation				S_F	S_G	S	
Rest of the World	M		L			O	
Government	T_I	T_B	B			T	
Households					P		W
Value added		V					

Table 2: Structure of Social Accounting matrix (Miller and Blair, 2009)

Matrix	Description
U	Total use of goods and services by industries
Q	Total income generated in economy
M	Import of goods and services from regions not included in supply and use table
X	Export of goods and services to regions not included in supply and use table
I	Investment in capital good
G	Government spending
F	Total use of goods and services by households
P	Government transfers to households (e.g. subsidies)
T	Direct taxation of consumers (e.g. income taxes)
T_I	Taxes on imported goods and services
T_B	Indirect taxes or taxes paid by businesses
S_F	Foreign savings
S_G	Taxes on imported goods and services
S	Household savings
D	Consumption of capital good (e.g. depreciation)
L	Lending of resources from regions not included in supply and use table
B	Government deficit spending
O	Transfer of money to regions not included in supply and use table
W	Income generated by households
H	Income generated by regions not included in supply and use table
V	Total use of Value Added by industries

Note that for Physical Supply and Use tables, some of the elements in Table 2.1 can initially be set to zero. That is, those that only refer to monetary flows or services. These include all taxes (T_I, T_B, T), all savings and lending, (S_F, S_G, S, L), wages (W), and transfers of money (P, O).

2.2 Imputation

In order to get to the best estimate of the PANORAMA database, first steps include gathering as much relevant data as possible and harmonizing that data. However, data collection is usually insufficient to produce a determinable system. A best first estimate for acknowledged data gaps should be utilized. This is called an imputation problem. Only after all data gaps are filled, can the database be balanced. This important step can help distinguish whether a zero in the final database represents no flow, or whether it is zero due to a lack of data.

Imputation measures for multi-regional SUT and IO tables in FIGARO distinguish between 'consistency' and 'manual' imputations (Remond-Tiedrez and Rueda-Cantuche (2019)). The former refers to data gaps where the total value of an item (e.g. country, industry or product) is known and only

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one sub-element is missing. Given this information, the value of the missing items can be systematically imputed (a determined system). Manual imputations require additional assumptions. For example, the production or use structure of a comparable country with a more detailed product or industry definition could be used for a country that only provides data on a more aggregated level.

At the moment this deliverable, the imputations of unobserved data for the PANORAMA project has not been fully established. This likely will differ according to the scale of the system.

2.3 Balancing properties

From a balancing perspective, the gapfilling procedure represents the initial guess value, which will be adjusted during the balancing, that is, adjusting the initial estimates such that they fulfil a determined set of criteria. For instance balancing a SAM (see Table 2.1) implies that the row sum equals the column sum. All conditions described in Section 2.1 need to be satisfied.

The literature review in Section 3 describes eleven approaches for balancing a Social Accounting Matrix (SAM). Each method begins with a short description of the underlying model followed by the desirable properties of each approach. The approaches are finally compared using the criteria below.

1. incorporate constraints on arbitrarily sized and shaped subsets of matrix elements, instead of only fixing row and column sums;
2. allow considering the reliability of the initial estimate;
3. be able to handle negative values and to preserve the sign of matrix elements if required;
4. be able to handle conflicting external data.
5. uses limited computational time.
6. is able balance a four-dimensional table.

The first four features are based on literature examples (Lenzen et al., 2009), whereas the last items are unique to PANORAMA. Each approach is held against these properties. While none of the 11 approaches is able to tick all the boxes, by prioritizing properties for a global physical stock flow table, the most suitable approach for PANORAMA can be chosen. An overview of all approaches and corresponding properties is discussed in Section 3.3. The highlights and important aspects of these features properties are as follows:.

The ability to *incorporate constraints on arbitrarily sized and shaped subsets of matrix elements* is convenient. Only fixing row sums or column sums of a SAM might not give enough information to find the best estimates for the interior. More information might be available, think about national bureau of statistics that provide supply and use tables on a more aggregated product and industry level. These are considered reliable data. The sum of products (industries) in the - to be balanced - SAM should add up to the aggregated product (industry) level of the national tables. Note that this turns the two dimensional balancing problem into a three-dimensional problem.

A robust balancing algorithm should consider *the reliability of initial estimates*. Ideally, an initial estimate of the interior SAM exists before starting the balancing procedure. This estimate includes IO elements from different sources and different quality. Gap filling procedures also make sure that estimates are given for missing data points. The latter gives a less reliable initial estimate compared to, for example, data points obtained from official bureau of statistics. Indicating which data points should stay more or less equal to its initial estimate is thereby an important property. Some methods define a vector of possible outcomes for each matrix element, this turns the two-dimensional balancing problem into a three-dimensional balancing problem.

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A stable balancing method should *handle negative values and preserve the sign of a matrix element*. That is, negative and positive matrix elements cannot switch signs. For some balancing methods it is impossible to switch signs. Other balancing methods stress that, if necessary, the approach should allow for switches of signs.

A balancing approach should be able to *handle conflicting information*. Fixed row and column sums are examples of external constraints. In combination with extra constraints on subsets of the matrix, constraints can be conflicting. Again a balancing routine should be flexible to allow for different reliability weights for each constraint. These constraints should not be relaxed. For other constraints, a level of flexibility (depending on its uncertainty) should be defined.

Although computational power is increasing rapidly, the *computational time* of finding the best estimate is also an important factor, especially for very large SAMs.

The sixth property, *being able to balance a four-dimensional table*, is an additional property required for the PANORAMA project. A regular SAM is a two-dimensional table. PANORAMA, on the other hand, aims to find best estimates for a three-dimensional table. That is, for each material element (e.g. cu, al, co) physical use and supply information is given in a separate matrix. The sum over the third dimension of the matrix adds another constraint. The matrix becomes four-dimensional when a balancing procedure should satisfy property 1 and 2.

3 Literature review of existing balancing methods

In the EXIOBASE string of projects, various balancing routines have been developed that allow detailing IOTs on the basis of auxiliary, more detailed information. In the PANORAMA project, we build upon this earlier developed knowledge. However, existing routines can be further extended and tailored for the current project. This document gives a thorough overview of existing balancing techniques.

This document makes the distinction in balancing methods by defining iterative and optimization methods. Bacharach has shown that these two type of approaches are in fact the same. It was shown that RAS, a well-known iterative balancing approach, can also be written as a constrained optimization problem. Other iterative balancing approaches are extensions of RAS.

The notations used for the different balancing procedures have been made uniform in the literature review. An overview of notations is given in the nomenclature in Appendix A. In general, a scalar is denoted in *italics* and a multi-dimensional object in **bold**. Lowercase denotes a vector and uppercase a matrix. Dimensions of a matrix element are given in subscripts. Superscripts denotes another version of the matrix. Superscript $*$ indicates it is the best estimate of that variable and superscript (0) indicates the initial guess of a variable. A vector placed between arrow brackets (<>) refers to a diagonalized vector.

3.1 Iterative methods

Iterative methods are used to solve a linear system problem by step wise getting closer to the final solution. Most iterative methods terminate when the residual value of the estimated matrix minus the matrix with target values is sufficiently small. This section discusses the RAS approach and its extensions. For each, a theoretical description of the method is given and its deviation from earlier defined iterative methods.

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3.1.1 RAS

The ultimate goal of balancing a square Social Accounting Matrix (SAM) is to find an interior that matches the sum of rows and columns. Naturally, the column sums should equal the row sums, because total produced is somewhere either consumed or stored. Assume existing SAM transaction matrix T, where the row and columns sums are not (yet) equal.

$$\sum_j t_{ij} \neq \sum_j t_{ji} \tag{1}$$

Based on most reliable raw input data, column and row sums should be fixed and equalized (resulting in vector $\mathbf{y}^*$). Now, the sum of the interior does not match the new row and column sums anymore. The goal is to find an interior that matches target vector $\mathbf{y}^*$.

Rather than optimizing matrix elements t_{ij}, often the elements a_{ij} of a corresponding matrix of coefficients A is being optimized. The relation between a_{ij} and t_{ij} is as follows:

$$t_{ij} = a_{ij} y_j. \tag{2}$$

The RAS methodology finds the optimal matrix of coefficients ($\mathbf{A}^*$) by means of biproportional row and column sum operations (Robinson et al. (2001)), such that

$$a^*_{ij} = r_i a_{ij} s_j. \tag{3}$$

Step 0: Initialize. Set n=0. Initial value of the coefficient matrix equals the existing matrix:
$\mathbf{A}^{*(\mathbf{0})} = \mathbf{A}$.

Step 1: Row scaling. Set
$n = n + 1$,
$\mathbf{R}^{*(n)} =< \mathbf{Ae} >< \mathbf{A}^{*(n-1)}\mathbf{e} >$.
$\mathbf{A}^{*(n-0.5)} = \mathbf{R}^{(n)}\mathbf{A}^{*(n-1)}$

Step 2: Column scaling. Set
$\mathbf{S}^{*(n)} =< \mathbf{e}'\mathbf{A} >< \mathbf{e}'\mathbf{A}^{*(n-0.5)} >$,
$\mathbf{A}^{*(n)} = \mathbf{A}^{*(n-0.5)}\mathbf{S}^{*(n)}$.

The advantage of RAS methodology is twofold. On the one hand it is a fairly easy approach that requires only a minimum amount of initial information. Only information on row and column sums is required, and an initial guess for the coefficients matrix. Also, negative values are not possible if they do not exist in the initial guess matrix.

This approach satisfies the following conditions from Section 2.3:

- be able to handle negative values and to preserve the sign of matrix elements if required.
- uses limited computational time.

3.1.2 MRAS

MRAS stands for <u>modified</u> RAS approach (Paelinck and Waelbroeck (1963)). The traditional RAS approach cannot deal with elements from coefficient matrix $\mathbf{A}$ that are certain. That is, to take the reliability of initial estimates into account. One would like to keep those elements fixed, while RAS methodology cannot make this guarantee.

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In the MRAS approach, elements from coefficient matrix $\mathbf{A}$ that are certain are set to zero in the interior, and are subtracted from the corresponding target column sum and row sum $\mathbf{y}^*$. This results in adjusted matrix $\mathbf{A}^{(0)}$ and alternative target row sum y_j^* and target column sum y_i^*. The traditional RAS methodology is applied to adjusted matrix $\mathbf{A}^{(0)}$, which ensures that zeros remain zero. The certain data points are introduced back into the balanced matrix. The final matrix is still balanced, because certain values had initially been subtracted from the target row and column sum.

Advantages of this approach over RAS is that it is able to deal with the reliability of the initial estimates. Disadvantage of this approach is that when too many sources are labeled as certain, only few elements in the matrix remain flexible for the RAS methodology. As a result, those elements will deviate extensively from their initial guess in matrix A, and thereby results in unreliable elements.

This approach satisfies the following conditions from Section 2.3:

- allow considering the reliability of the initial estimate;
- be able to handle negative values and to preserve the sign of matrix elements if required.
- uses limited computational time.

3.1.3 ERAS

Similar to the MRAS approach, in the ERAS approach interior values of the forecasted matrix are fixed to known values (Lahr and de Mesnard (2004)). This approach has been developed in the unpublished PhD dissertation of Philip Israilevich "a Biproportional Forecasting of Input-Output Tables". Israilevich was the PhD student of Ronald E. Miller and Peter Blair. Little information can be found online on this approach, which makes this approach unsuitable for the Panorama project. We have added this approach in this literature research for the sake of completeness.

This approach satisfies the following conditions from Section 2.3:

1. allow considering the reliability of the initial estimate;
2. be able to handle negative values and to preserve the sign of matrix elements if required;
3. uses limited computational time.

3.1.4 TRAS

TRAS stands for three-staged RAS approach (Gilchrist and St Louis (1999)). This approach extends RAS and MRAS, because it allows to fix certain aggregated information on submatrices of the SAM, rather than just individual elements (like MRAS).

Assume that some elements of Social-Accounting matrix $\mathbf{T}$ are certain. These elements are placed in matrix $\mathbf{T^C}$ which has the same size as $\mathbf{T}$. Certain values are set to zero in the initial table which is used for the RAS procedure, denote this table by $\mathbf{T}^0$. The elements in $\mathbf{T^C}$ are also subtracted from the corresponding target column sum and row sum $\mathbf{y}^*$ (so far the same as using MRAS). Then the iterative RAS approach is used for the balancing, however, with one addition. The TRAS approach adds a thirds step to the two-step RAS approach where it allows to place a constrained on a subset of matrix elements.

Assume exogenous aggregate table $\mathbf{T^G}$, which has for example a less detailed product and sector definition. However, the aggregate values in this table are considered trustworthy. Summation of an optimized SAM $\mathbf{T}^*$ should therefore add up to the values in table $\mathbf{T^G}$. Let $\mathbf{U}$ and $\mathbf{V}$ represent the known row and column aggregator matrices. For best estimate $\mathbf{T}^*$ it should hold that

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$$\mathbf{T^G} = \mathbf{U}\mathbf{T}^*\mathbf{V}. \tag{4}$$

Then, the certain individual elements in matrix T^C should be consistent with the aggregates in matrix $\mathbf{T^G}$. Define table $\mathbf{T^{GC}} = \mathbf{T^G} - \mathbf{U}\mathbf{T^C}\mathbf{V}$, which gives the aggregates of the unknown cells of $\mathbf{T^C}$. In the RAS procedure, best estimate $\mathbf{T}^{T*}$ is found, where $\mathbf{T}^* = \mathbf{T}^{T*} + \mathbf{T^C}$. Optimized table $\mathbf{Q}^*$ should fulfill the following condition for the third iteration step

$$\mathbf{T}^{GC} = \mathbf{U}\mathbf{T}^{T*}\mathbf{V}. \tag{5}$$

In summary, including the third iteration step, the procedure for TRAS is given by the following three steps:

Step 0: Initialize. Set n=0. Initial value of the coefficient matrix equals the existing matrix:
$$\mathbf{A}^{*(0)} = \mathbf{A}.$$

Step 1: Row scaling. Set
$$n = n + 1,$$
$$\mathbf{R}^{*(n)} = <\mathbf{Ae}><\mathbf{A}^{*(n-1)}\mathbf{e}>,$$
$$\mathbf{A}^{*(n-0.5)} = \mathbf{R}^{(n)}\mathbf{A}^{*(n-1)}$$

Step 2: Column scaling. Set
$$\mathbf{S}^{*(n)} = <\mathbf{e}'\mathbf{A}><\mathbf{e}'\mathbf{A}^{*(n-0.5)}>,$$
$$\mathbf{A}^{*(n-0.25)} = \mathbf{A}^{*(n-0.5)}\mathbf{S}^{*(n)}.$$

Step 3: incorporate aggregate matrix $\mathbf{A^G}$. Set
$$\mathbf{G}^0 = \mathbf{A^{GC}} \oslash \mathbf{U}\mathbf{A}^{*(n-0.25)}\mathbf{V},$$
$$\mathbf{Q}^{*(n)} = \bar{\mathbf{U}}\mathbf{G}^0\bar{\mathbf{V}},$$
$$\mathbf{A}^{*(n)} = \mathbf{Q}^{*(n)} \circ \mathbf{A}^{*(n-0.25)}.$$

Again, angled brackets <> denote a diagonal matrix, where vector elements are presented on the diagonal and zeros elsewhere, e denotes a vector with ones. In Step 3, $\oslash$ denotes an element wise division (Hadamard division), $\circ$ and element wise product (Hadamard product), $\mathbf{A^{GC}}$ denotes the coefficient matrix of matrix $\mathbf{T^{GC}}$, and $\bar{\mathbf{U}}$ and $\bar{\mathbf{V}}$ transform matrix $\mathbf{Q}^*$ to the original size of matrix $\mathbf{A}$.

This approach satisfies the following conditions from Section 2.3:

1. incorporate constraints on arbitrarily sized and shaped subsets of matrix elements, instead of only fixing row and column sums;
2. allow considering the reliability of the initial estimate;
3. be able to handle negative values and to preserve the sign of matrix elements if required;

3.1.5 GRAS

GRAS stands for generalized RAS approach (Junius and Oosterhaven (2003)). This approach deals with negative values in a SAM. A more simple approach to deal with negative values is to assume that negative values are certain and apply the MRAS approach. That is, set the negative values to zero in the social accounting matrix, resulting in initial matrix $\mathbf{T}^{(0)}$ and subtract the negative values from the target row and column sum. Place the negative values in matrix $\mathbf{T^C}$. The RAS procedure is applied to coefficient matrix $\mathbf{A^0}$ of SAM $\mathbf{T^0}$. Disadvantage of this approach is that negative values are fixed and cannot positively or negatively affect the other elements in the SAM, they are in fact ignored. Junius and Oosterhaven (2003) propose a different approach for dealing with negative numbers. It proposes to include absolute values for negative elements in the objective

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function. For this, matrix $\mathbf{A}$ is split in a positive part ($\mathbf{P}$) and a negative part ($\mathbf{N}$). Difference $\mathbf{A} = \mathbf{P} - \mathbf{N}$ is balanced, where it should hold that

$$(\mathbf{RPS} - \mathbf{R}^{-1}\mathbf{NS}^{-1})\mathbf{i} = e\mathbf{y_j}^*, \tag{6}$$

$$\mathbf{i}(\mathbf{RPS} - \mathbf{R}^{-1}\mathbf{NS}^{-1}) = e\mathbf{y_i}^*, \tag{7}$$

where $\mathbf{y_j}^*$ the vector with row sums, $\mathbf{y_i}^*$ the vector with column sums, e the basis of a natural logarithm, and $\mathbf{i}$ is the summation vector. Below, the iteration process has been described:

Step 0: Initialize. Set n=0. Initial value of the coefficient matrix equals the existing matrix:
$\mathbf{A}^{*(0)} = \mathbf{A}$.
$\mathbf{R}^{*(0)} = \mathbf{i}$

Step 1: Column scaling. Set
$n = n + 1$,
$\mathbf{p}_j(\mathbf{R}^{*(n-1)}) = \sum_i p_{ij}\mathbf{R}_i^{r(n-1)}$
$\mathbf{n}_j(\mathbf{R}^{*(n-1)}) = \sum_i n_{ij}\mathbf{R}_i^{r(n-1)}$
$\mathbf{S}_j^{*(n)} = \frac{y_{jc}^* \sqrt{(y_{jc}^*)^2 + 4\mathbf{p}_j(\mathbf{R}^{*(n-1)})\mathbf{n}_j(\mathbf{R}^{*(n-1)})}}{2\mathbf{p}_j(\mathbf{R}^{*(n-1)})}$

Step 2: Row scaling. Set
$\mathbf{p}_i(\mathbf{S}^{*(n)}) = \sum_j p_{ij}\mathbf{S}_j^{*(n)}$
$\mathbf{n}_i(\mathbf{S}^{*(n)}) = \sum_j n_{ij}\mathbf{S}_j^{*(n)}$
$\mathbf{R}_i^{*(n)} = \frac{y_{ir}^* \sqrt{(y_{ir}^*)^2 + 4\mathbf{p}_i(\mathbf{S}^{*(n)})n_i(\mathbf{S}^{*(n)})}}{2\mathbf{p}_i(\mathbf{S}^{*(n)})}$
$\mathbf{A}^{*(n)} = \mathbf{R}^{*n}\mathbf{A}^{*(n-0.5)}$

Note the difference between iteration counter n and element from matrix $\mathbf{N}$, i.e. n_{ij}.

This approach satisfies the following conditions from Section 2.3:

1. incorporate constraints on arbitrarily sized and shaped subsets of matrix elements, instead of only fixing row and column sums;
2. allow considering the reliability of the initial estimate;
3. be able to handle negative values and to preserve the sign of matrix elements if required;

3.1.6 KRAS

KRAS stands for Konfliktfreies RAS approach (Lenzen et al. (2009)). This approach is an extension of the GRAS approach. In addition, it is able to balance SAMs under conflicting external information and inconsistent constraints.
Assume a set of constraints. This might include row and column sum constraints, single elements that are considered certain, constrained subsets of the SAM, preserving the negative sign of elements. These constraints are gathered in the formulation

$$\mathbf{Ga} = \mathbf{c}. \tag{8}$$

where $\mathbf{a}$ is the vectorization of coefficient matrix $\mathbf{A}$. That is, square SAM matrix $\mathbf{A}$ consisting of m rows and m columns is converted to one vector of size $m \cdot m$. Matrix $\mathbf{G}$ gives information on the sum of elements of $\mathbf{a}$ that should add up to the corresponding element in $\mathbf{c}$. The number of rows in $\mathbf{G}$ indicate the number of constraints. We assume $\mathbf{G}$ has n_c rows.
Under the GRAS approach, the iteration stops when $\mathbf{Ga}$ comes sufficiently close to $\mathbf{c}$, i.e.

$$\|\mathbf{Ga} - \mathbf{c}\| < \delta\|\mathbf{c}\| \tag{9}$$

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for a sufficiently small δ. However, in case some of the constraints are conflicting, the algorithm will not find a solution that satisfies all constraints. Adding extra iterations, does not improve the distance between $\mathbf{c}$ and $\mathbf{Ga}$ anymore. In this situation, GRAS algorithm allows for adjustment of the constraint, namely the level of $\mathbf{c}$. An amount $\alpha\sigma_k$ can be added or subtracted to constraint c_k for all constraints $k = \{1, \cdots n_c\}$. Scalar $0 \leq \alpha \leq 1$ is free of choice and σ_k corresponds to predefined standard deviations of c_k. In the GRAS approach, there is one scalar for each constraint k. This approach iteratively updates scalar r_k and vector element a_l. Where l indicates the placing in vectorized a. The GRAS iteration process is described below:

Step 0: Initialize. Set n=0. Initial value of the coefficient matrix equals the existing matrix:

$$\mathbf{A}^{*(0)} = \mathbf{A}.$$
$$c_k^{(0)} = c_k$$

Step 1: Updating of scalars r and c

$$r_k^{(n)} = \frac{c_k^n + \sqrt{(c_k^{(n)})^2 + 4\sum_l g_{kl}^+ a_l^{n-1} \sum_l g_{kl}^- a_l^{n-1}}}{2\sum_l g_{kl}^+ a_l^{n-1}}$$
$$c_k^{(n)} = c_k^{(n-1)} - sgn(c_k^{(n-1)} - \textstyle\sum_l g_{kl} a_l^{(n-1)}) \cdot \min(|c_k^{(n-1)} - \sum_l g_{kl} a_l^{(n-1)}|, \alpha\sigma_k)$$

Step 2: Updating vectorized matrix a

$$a_l^{(n)} = a_l^{(n-1)} (r^{(n)})^{sgn(g_{kl})}$$

Element g_{kl}^+ refers to positive values of g_{kl} and g_{kl}^- refers to negative elements.

This approach satisfies the following conditions from Section 2.3:

- incorporate constraints on arbitrarily sized and shaped subsets of matrix elements, instead of only fixing row and column sums;
- allow considering the reliability of the initial estimate;
- be able to handle negative values and to preserve the sign of matrix elements if required;
- be able to handle conflicting external data.

3.2 Constrained optimization

Besides iterative methods, constrained optimization methods also aim at solving balancing problems. In general, the most basic balancing problem solved using constrained optimization is given by

$$\min f(\mathbf{A}, \mathbf{A}^0) \qquad s.t. \qquad \sum_j a_{ij} y_j = y_i, \qquad \sum_i a_{ij} = 1, \qquad a_{ij} \geq 0. \tag{10}$$

Objective function $f(\cdot)$ is minimized under row-sum and columns-sum constraints. In this section we review a couple of constrained optimization methods.

3.2.1 Cross-entropy method

Iterative RAS approach and constrained optimization approaches are relatively close to each other. Bacharach shows that the conventional RAS methodology can be written as a constrained optimization problem, where the objection function is given by

$$f(\mathbf{A}, \mathbf{A}^{(0)}) = \sum_{ij} a_{ij} ln(\frac{a_{ij}}{a_{ij}^{(0)}}) \tag{11}$$

This objective function is also known as the cross-entropy (CE) function. Matrix $\mathbf{A}^{(0)}$ is an initial guess for best estimate $\mathbf{A}^*$. A SAM from a previous period can be used as first guess (Golan

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and Vogel (2000)). In constrained optimization problems Lagrangian L is defined and minimized in order to find the technical coefficients a_{ij} that minimizes the cross-entropy function. In this approach coefficients a_{ij} are treated as probabilities, and the $\mathbf{A}$ is viewed as probability distribution. The cross-entropy minimization gives estimates for a posterior probability distribution ($\mathbf{A}^*$) which is closest to prior information ($\mathbf{A}^{(0)}$) Rodrigues (2014)). Lagrangian L of equation (10) and (11) is defined by

$$L = f(\mathbf{A}, \mathbf{A}^{(0)}) + \sum_i \lambda_i (y_i - \sum_j a_{ij} y_j) + \sum_j \mu_j (1 - \sum_i a_{ij}), \tag{12}$$

and estimated technical coefficients after optimization are given by:

$$a^*_{ij} = \frac{a^{(0)}_{ij} \exp(\lambda^*_i y_j)}{\sum_i a^{(0)}_{ij} \exp(\lambda^*_i y_j)}. \tag{13}$$

Robinson et al. (2001) compare this expression to Bayes' Theorem, where posterior (a_{ij}) equals prior ($a^{(0)}_{ij}$) multiplied by the likelihood function divided by a normalization factor. The likelihood function expresses the probability of drawing the data given estimated parameters.

Up to this point, the CE-estimator is equal to the basic RAS procedure. Only information on row and column sums is required. Robinson et al. (2001) point out that CE-estimation is able to include more information in the constraints:

- Economic aggregates. For example, exogenous information on aggregated subsets of products and sectors. Assume aggregation matrix G that gives information on the sum of elements in T that should add up to aggregate γ. Element g_{ij} is equal to 1 if corresponding SAM-element t_{ij} is part of the aggregation, and zero otherwise. Assume k constraints, all constraints can be added to objective function in equation (10). Constraint k is given by:

$$\sum_i \sum_j g^{(k)}_{ij} t_{ij} = c_k. \tag{14}$$

  Note that the column sum and row sum conditions are a special case of this formulation.

- Uncertainty in aggregates. One way to solve uncertainty in the aggregates in equation (14) is by setting an upper or lower bound on the aggregate value. This is done by replacing the equal-sign to an inequality sign:

$$\sum_i \sum_j g^{(k)}_{ij} t_{ij} \leq c_k \qquad \text{or} \qquad \sum_i \sum_j g^{(k)}_{ij} t_{ij} \geq c_k. \tag{15}$$

- Zeros. The RAS approach ensures that zeros in the initial matrix remain zero in the best estimate matrix. A similar assumption can be posed for the CE-approach. For this, make use of the property $x \ln(x) = 0$. In the current objective function, initial estimates equal to zero imply division by zero. By replacing $a^{(0)}_{ij}$ with $a^{(0)}_{ij} + \delta$ and a_{ij} with $a_{ij} + \delta$ (δ is sufficiently small) this problem is overcome and a^*_{ij} can be equal to zero.

This approach satisfies the following conditions from Section 2.3:

- incorporate constraints on arbitrarily sized and shaped subsets of matrix elements, instead of only fixing row and column sums;
- allow considering the reliability of the initial estimate;
- be able to handle negative values and to preserve the sign of matrix elements if required;

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3.2.2 Maximum entropy method

The main difference between the maximum entropy (ME) and the cross-entropy (CE) method is that CE takes prior information on the initial estimates $a_{ij}^{(0)}$ into account. This prior information could be taken from a SAM of a previous year. ME does not make use of prior information. That is, $a_{ij}^{(0)} = 1/m$ for a square coefficient matrix $\mathbf{A}$ of SAM $\mathbf{T}$, that has dimension m by m (Golan and Vogel (2000)).

The objective function of ME-estimation looks as follows

$$f(\mathbf{A}, \mathbf{A}^{(0)}) = \sum_{ij} a_{ij} \ln(a_{ij}). \tag{16}$$

Replacing $a_{ij}^{(0)} = 1/m$ in equation (13) gives the optimal estimates in ME-optimization. Note that CE-estimation is a generalization of ME-estimation. The extra information that can be included for CE, can also be added to the optimization function for ME.

This approach satisfies the following conditions from Section 2.3:

- incorporate constraints on arbitrarily sized and shaped subsets of matrix elements, instead of only fixing row and column sums;
- allow considering the reliability of the initial estimate;
- be able to handle negative values and to preserve the sign of matrix elements if required;

3.2.3 General Cross-Entropy method

Golan and Vogel (2000) provides a variation of the traditional cross-entropy method, referred to as the general cross-entropy (CE) method. The general CE optimizes transaction values (t_{ij}) rather than coefficients (a_{ij}).

Also, traditional CE treats coefficients a_{ij} as probabilities, and coefficient matrix $\mathbf{A}$ as probability distribution. General CE treats each element of SAM (t_{ij}) as a random variable that can take M possible values: $b_{ij} = [b_{ij1}, \cdots, b_{ij}^*, \cdots, b_{ijM}]$ and is centered around $\mathbf{b_{ij}}^*$. These support vectors can be different for every element in the SAM. For b_{ij}^* it holds that $b_{ij}^* = t_{ij}^{(0)}$, where $t_{ij}^{(0)}$ the corresponding element in a first guess for $\mathbf{T}$, which could be taken from a SAM of an earlier year. For $M = 3$, b_{ij} could have the following values $[t_{ij}^{(0)}(1-r), t_{ij}^{(0)}, t_{ij}^{(0)}(1+r)]$. A relatively high value of scalar r represents a situation of wide boundaries between which the best estimate could be placed. A switch of sign is allowed for $|r| > 1$. For example, a positive value of initial element $t_{ij}^{(0)}$, can include negative values in its corresponding support vector. This might result in a negative best estimate t_{ij}^* (Fernandez-Vazquez (2016)).

Each support vector $\mathbf{b_{ij}}$ is connected to a symmetric vector with probabilities, $\mathbf{q_{ij}} = [q_{ij1}, \cdots, q_{ij}^*, \cdots, q_{ijM}]$. A simple example for situation $M = 3$ is given by $\mathbf{q_{ij}} = [\frac{1}{3}, \frac{1}{3}, \frac{1}{3}]$, which placed equal probabilities to each of the possible outcomes in $\mathbf{b_{ij}}$. A spiky distribution like $\mathbf{q_{ij}} = [0.05, 0.9, 0.05]$ can be used when initial element $t_{ij}^{(0)}$ is considered certain. Under these conditions, elements in the initial matrix $T^{(0)}$ are equal to the expected value described in equation (17).

$$t_{ij}^{(0)} = \sum_{m=1}^{M} q_{ijm} b_{ijm}. \tag{17}$$

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For target matrix $\mathbf{T}^*$ (also known as 'best estimate'), the probabilities are unknown. The goal is to find (posterior) probabilities $\mathbf{p_{ij}} = [p_{ij1}, \cdots, p^*_{ij}, \cdots, p_{ijM}]$ that satisfy

$$t^*_{ij} = \sum_{m=1}^{M} p_{ijm} b_{ijm}, \tag{18}$$

where $\mathbf{q_{ij}}$ can be taken as an prior for $\mathbf{p_{ij}}$.

Under classical CE, the objective function is optimized for coefficient a_{ij} (see equation (11)). For General CE, the objective function is given by

$$f(\mathbf{P}, \mathbf{Q}) = \sum_{ijm} p_{ijm} ln(\frac{p_{ijm}}{q_{ij}}), \tag{19}$$

subject to the following conditions

$$\sum_{j} t^*_{ij} = \sum_{jm} p_{ijm} b_{ijm} = y_i, \tag{20}$$

$$\sum_{i} t^*_{ij} = \sum_{im} p_{ijm} b_{ijm} = y_j, \tag{21}$$

where y_i and y_j represent the column and row sum respectively. Following the derivations of Lagrange optimization in Fernandez-Vazquez (2016), it is found that best estimate for the probability matrix P is given by

$$p_{ijm} = \exp(\pi_i b_{ijm}) q_{ijm} \exp(\lambda_i b_{ijm} - 1), \tag{22}$$

where π_i and λ_i denote the Lagrange multipliers that correspond to equation (20) and (21) respectively.

Scandizzo and Ferrarese (2015) applies the described approach to find the best estimate for SAM of the Italian economy. Scandizzo and Ferrarese (2015) extends GCE by including Monte Carlo bootstrap estimates of the probability distribution of all SAM parameters. This allows to include exogenous information from for example time series of national accounts. Also, it takes historical volatility of the main variables into account. For each set of parameters the SAM is balanced and a best estimate is found. The average of all best estimates gives a final best estimate of the SAM of the Italian economy.

One of the advantages of this approach is that it allows for switching of signs, if required.

This approach satisfies the following conditions from Section 2.3:

- incorporate constraints on arbitrarily sized and shaped subsets of matrix elements, instead of only fixing row and column sums;
- allow considering the reliability of the initial estimate;
- be able to handle negative values and to preserve the sign of matrix elements if required;

3.2.4 (Generalized) Least Square Method

Least-Square method is an optimization approach that replaced the objective function of the general cross-entropy method by

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$$f(\mathbf{t}, \mathbf{t}^0, \sigma) = (\mathbf{t} - \mathbf{t}^0)' \Sigma_{t^0}^{-2} (\mathbf{t} - \mathbf{t}^0), \tag{23}$$

where $\mathbf{t}$ denotes the vectorization of square coefficient matrix $\mathbf{T}$, $\mathbf{\Sigma_t^{-2}}$ denotes a diagonal weight matrix with variances on the diagonals, $< \sigma > < \sigma >$ (Geschke et al.). Angled brackets $<>$ denote a diagonal matrix, and σ is a vector with standard deviations σ_j corresponding to each column j. Note, this implies that best estimate $\mathbf{T}^*$ is determined given not only best guess $\mathbf{T}^0$, but also given an uncertainty parameter. All constraints are collected in matrix multiplication

$$\mathbf{G}\mathbf{t} = \mathbf{c}, \tag{24}$$

where $\mathbf{t}$ is the vectorization of SAM $\mathbf{T}$. That is, SAM Matrix $\mathbf{T}$ consists of m rows and columns, and is converted to one vector of size $m \cdot m$. Matrix $\mathbf{G}$ gives information on the sum of elements of $\mathbf{t}$ that should add up to the corresponding element in $\mathbf{c}$. Setting up the Lagrangian gives best estimate

$$\mathbf{t}^* = \mathbf{t}^0 + \mathbf{\Sigma_{t^0}^{-2}} \mathbf{G}' \alpha^*, \tag{25}$$

where α equals the first moment of the Lagrangian multiplier λ. Best guess for α^* is the solution of

$$(\mathbf{GSG}')\alpha^* = -(\mathbf{G}\mathbf{t}^0 + \mathbf{t}^*). \tag{26}$$

The Least-Squares approach can be extended in a number of ways. In this section we extend the approach by considering the reliability of initial estimates and dealing with conflicting information. This extension of the Least-Square Method is described in Geschke et al., which relies on the approach developed by van der Ploeg (1982) and Van der Ploeg (1988). The approach allows for the distinction of two types of constraints: hard and soft constraints. Assume $\mathbf{H}$ hard and $\mathbf{S}$ soft constraints are defined. Restructuring and prioritization of the constraints in equation 24 gives

$$\begin{pmatrix} \mathbf{G_{hard}} \\ \mathbf{G_{soft}} \end{pmatrix} t = \begin{pmatrix} \mathbf{c_{hard}} \\ \mathbf{c_{soft}} \end{pmatrix}. \tag{27}$$

The hard constraints should always hold, however the soft constraints can be violated. Assume $\mathbf{g_{soft,k}}$ is one row of $\mathbf{G_{soft}}$ that adds up to $c_{soft,k}$. Violation of constraint k is given by error term ϵ_k, such that

$$\mathbf{g_{soft,k}} \cdot \mathbf{t} = c_{soft,k} + \epsilon_k. \tag{28}$$

Geschke et al. describes the intermediate derivations that give final constraint matrix G^{GLS}:

$$\begin{pmatrix} \mathbf{G_{hard}} & 0 \\ \mathbf{G_{soft}} & -\mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{t} \\ \epsilon \end{pmatrix} = \begin{pmatrix} \mathbf{c_{hard}} \\ \mathbf{c_{soft}} \end{pmatrix}. \tag{29}$$

That is,

$$\mathbf{G^{GLS}} \mathbf{t^{GLS}} = \mathbf{c^{GLS}}, \tag{30}$$

where

$$\mathbf{G^{GLS}} = \begin{pmatrix} \mathbf{G_{hard}} & 0 \\ \mathbf{G_{soft}} & -\mathbf{I} \end{pmatrix}, \qquad \mathbf{t^{GLS}} = \begin{pmatrix} \mathbf{t} \\ \epsilon \end{pmatrix}, \qquad \text{and} \qquad \mathbf{c^{GLS}} = \begin{pmatrix} \mathbf{c_{hard}} \\ \mathbf{c_{soft}} \end{pmatrix}. \tag{31}$$

By optimizing Equation 23 under assumption of constraints in Equation 30, optimal estimates for $\mathbf{T}^*$ are found. This approach takes into account the reliability of the initial estimates via definition of soft and hard constraints, and deals with conflicting constraints via flexibility in $\mathbf{C^{GLS}}$.

Other extensions of the LS method are possible. Rampa (2008) extends the classical LS procedure with a 'subjective' variant. This procedure is flexible in the sense that it column and row sums should not necessarily be given as exogenous, it allows for adding extra constraints. Also it considers the reliability of the initial estimates by attaching weights to the 'first guess' matrix that should be balanced.

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This approach satisfies the following conditions from Section 2.3:

- Incorporate constraints on arbitrarily sized and shaped subsets of matrix elements.
- Consider the reliability of the initial estimate.
- Is able to handle negative values and is able to preserve the sign of matrix elements if required.
- Is able to handle conflicting information.

3.2.5 Linear method (Rodrigues (2014))

Rodrigues (2014) reports a sequence of methods that are derived by simplifying an ideal data problem. The starting point is a formalization of the data balancing problem as a set of accounting identities, $\mathbf{G}$, that linearly connect variables $\mathbf{t}$, and constraints, $\mathbf{k}$. Then the numbers reported in statistical data are interpreted as an expected value of an underlying probability distribution of which uncertainty information can be used to estimate second moments (covariances). Relative cross-entropy minimization is applied to obtain a distribution whose first and second moments satisfy all accounting identities. Because second moment data is scarce, the method recommended for actual implementation assumes that only variances but not off-diagonal covariances are known, leading to the linear method, which is in practice an iterative weighted least squares method, in which coefficients of variation (and not variances) are used as weighing factors. This choice is justified by the interpretation that correlations are implicitly assumed to be close to unitary such that the coefficient of variation of disaggregate and aggregate data is similar (an accounting identity typically connects one aggregate datum to multiple disaggregate data). The linear method works as follows.

A balanced system, $\mathbf{t}_\infty$, would satisfy:

$$\mathbf{G}\mathbf{t}_\infty = \mathbf{k}$$

whereas the initial estimate, $\mathbf{t}_0$, instead satisfies:

$$\mathbf{G}\mathbf{t}_0 \neq \mathbf{k}$$

For iteration $k \geq 0$ the update rule is:

$$\mathbf{t}_{k+1} = \mathbf{t}_k + \delta \hat{\mathbf{u}}\hat{\mathbf{t}}_k \mathbf{G}'\boldsymbol{\alpha}$$

In turn the vector of Lagrange multipliers, $\boldsymbol{\alpha}$, is determined as the solution of:

$$\left(\mathbf{G}\hat{\mathbf{u}}\hat{\mathbf{t}_k}\right)\mathbf{G}'\boldsymbol{\alpha} = \mathbf{k} - \mathbf{G}\mathbf{t}_k$$

There might be a unique solution, otherwise a pseudo-inverse can be calculated. Finally, the adjustment step δ is calculated so that the relative adjustment is small for every variable.

$$\mathbf{t}^*_{k+1} = \mathbf{t}_k + \hat{\mathbf{u}}\hat{\mathbf{t}}_k \mathbf{G}'\boldsymbol{\alpha}$$
$$\delta = \min\left\{1, \epsilon \left| \frac{t_{i(k+1)}}{t^*_{i(k)} - t_{i(k)}} \right|_i \right\}$$

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where ϵ is a small number. This procedure assumes that all variables are non-zero by definition, implying that empties are excluded from the set of variables. This procedure also implies that variables are not allowed to shift signs: if variables are meant to be allowed to shift signs then they should be excluded from the set of variables used to determine δ.

The paper also considers that besides uncertainty information, which is summarized in vector $\mathbf{u}$ (containing positive numbers, usually in the range from zero to one), the source data might also be split into a hierarchy of data quality, such that higher-level data is kept constant while lower-level data is adjusted sequentially.

The computational speed of Rodrigues (2014) can be further improved by parallel implementation of the balancing algorithm as is shown in Silva (2015).

This approach satisfies the following conditions from Section 2.3:

- Incorporate constraints on arbitrarily sized and shaped subsets of matrix elements.
- Consider the reliability of the initial estimate.
- Is able to handle negative values and is able to preserve the sign of matrix elements if required.
- Is able to handle conflicting information.

3.3 Discussion and Conclusions

This section discusses all balancing approaches and corresponding properties. An overview is given in Table 3.3. Let us recall the six desirable properties - described in Section 2.3 - that an ideal balancing approach could fulfill:

1. incorporate constraints on arbitrarily sized and shaped subsets of matrix elements, instead of only fixing row and column sums;
2. allow considering the reliability of the initial estimate;
3. be able to handle negative values and to preserve the sign of matrix elements if required;
4. be able to handle conflicting external data;
5. uses limited computational time;
6. is able balance a four-dimensional table.

The only methods that are able to handle conflicting information are KRAS, General Cross-Entropy procedure and the linear approach. As expected, there is no approach that is able to balance a three-dimensional table. The balancing procedure choice for PANORAMA is an extension of the linear approach. This choice is made for the following reasons. (1) it belongs to one of the three procedures that satisfies most properties (2) contrary to GLS, the linear approach is solved in an iterative manner, which gives insight in the path towards solution (3) the founder of this approach is part of the PANORAMA team, which makes this approach the easiest to extend.

Table 3: Table with properties corresponding to balancing procedures

	RAS	MRAS	ERAS	TRAS	GRAS	KRAS	CE	ME	GCE	GLS	LA
1				X	X	X	X	X	X	X	X
2		X	X	X	X	X	X	X	X	X	X
3	X	X	X	X	X	X	X	X	X	X	X
4						X				X	X
5	X	X	X								
6											

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